RAMAKRISHNA MISSION VIDYAMANDIRA

(A Residential Autonomous College under University of Calcutta)

First Year, Second Semester (January – June), 2011

Mid-Semester Examination, March, 2011

MATHEMATICS (General)

Date : 12 March 2011 Full Marks : 25

Time : 11am – 12noon

Group – A

1. Answer **either (a) or (b)** : [5]

   a) Show that the lines joining the origin to the points of intersection of the parabola $y^2 = 4ax$ and the straight line $y = mx + c$ are

      i) at right angles if $c + 4am = 0$,

      ii) Coincident if $c = \frac{a}{m}$.

   b) The latus rectum of a conic is 6 and its eccentricity is $\frac{1}{2}$. Find the length of the focal chord inclined at an angle 60° with the major axis.

Group – B

2. Answer **either (a) or (b)** : [5]

   a) i) Prove by vector method that the medians of a triangle are concurrent.

      ii) Find the unit vector parallel to the resultant of the vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$. [3+2]

   b) i) In a triangle ABC, prove that $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$, where a, b, c; A, B, C have usual meaning.

      ii) If two vectors $\vec{a}, \vec{b}$ are such that $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, find the angle between $\vec{a}$ and $\vec{b}$. [3+2]

Group – C

3. Answer **any two** questions : [4+4 = 8]

   a) Define a bounded sequence with an example. Prove that every convergent sequence is bounded. [(1+1)+2]

   b) Prove that the sequence $\{x_n\}$ where $x_n = 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...+\frac{1}{2^n}$ is convergent. Also find the limit of the sequence. [3+1]

   c) If the series $\sum_{n=1}^{\infty} u_n$ is convergent, prove that $\lim_{n\to\infty} u_n = 0$.

   Prove that the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+...$ is convergent. [2+2]

Group – D

4. Answer **any one** question : [4]

a) $I_n = \int_0^{\pi/4} \tan^n x\,dx$, where n is a positive integer (≥ 2), show that $I_n = \frac{1}{n-1} - I_{n-2}$. Use this result to evaluate $\int_0^{\pi/4} \tan^6 x\,dx$. [2+2]

b) If $I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x\,dx$, where m and n are natural numbers; show that $I_{m,n} = \frac{m-1}{m+n} I_{m-2,n}$ where $m > 2$. [4]

Group – E

5. Answer **any one** question : [3]

a) Find the necessary and sufficient condition for the ordinary differential equation Mdx + Ndy = 0 to be exact.

b) What is meant by Integrating factors? Describe any one rule for finding the integrating factor.